3D Printing the Trefoil KNOT and its Pages.

2015 UGA CURO Symposium
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Topics.

- The Question
- Knot Examples
- 3D Mesh Manipulation
- 3D Printing
- Photo Gallery
- Results
- Final Remarks and Questions

The Question.

- Given a **complicated** shape defined **mathematically**, what is the best way to create a **puzzle** from the shape demonstrating its **properties**?

- Construction of object.

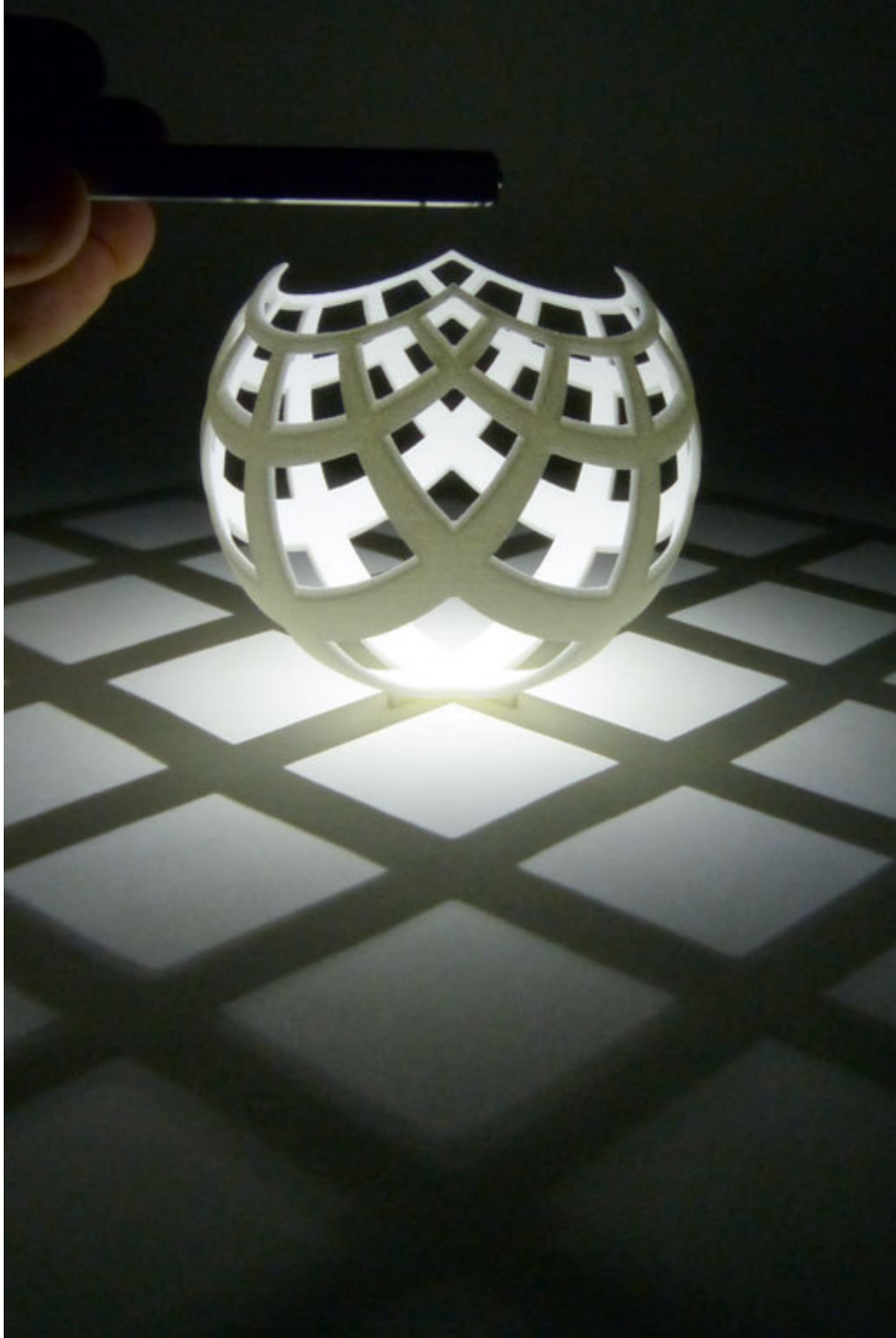
    1. Take the inverse image of sets in complex plane

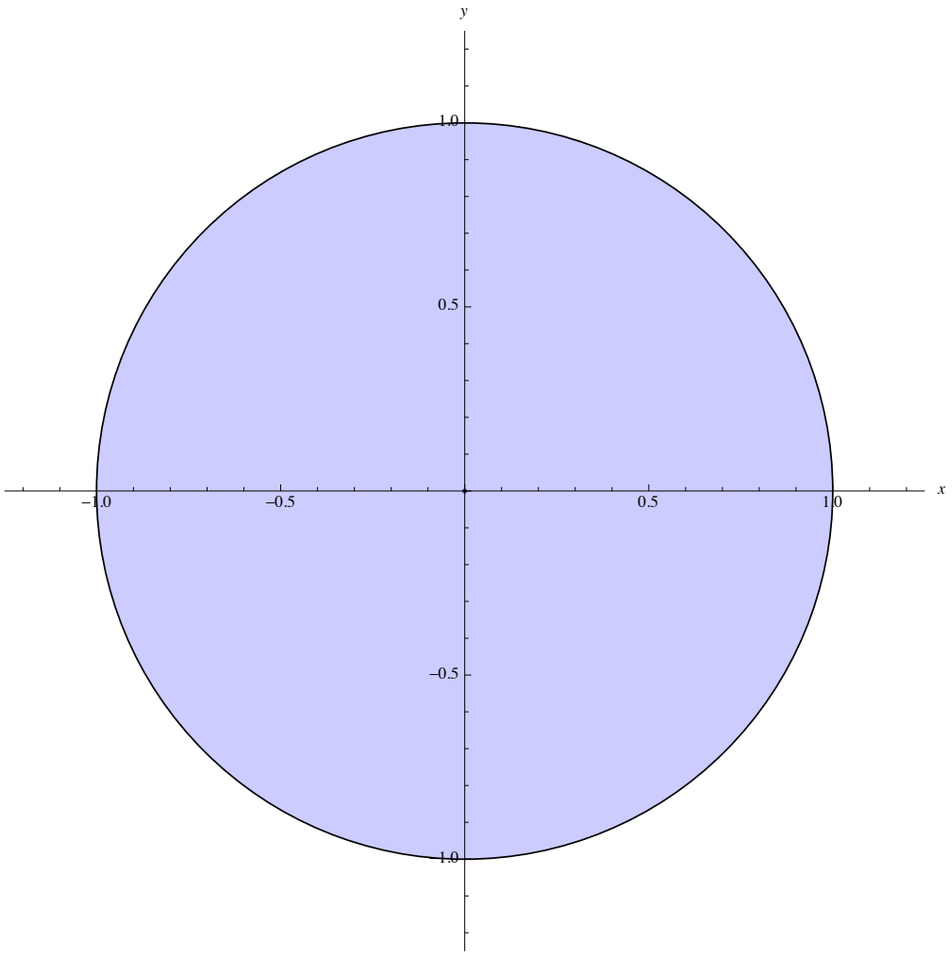
    2. Stereographic project into 3D from 4D

- In other words, composition of two functions.

Theory.

- **Stereographic Projection**: Mapping from the sphere in 3D to the plane in 2D
- Now generalize: project from the "sphere" in 4D ($\mathbb{C}^2 = \mathbb{R}^4$) onto $\mathbb{R}^3$
- **Open-book decomposition**: imagine our knot is the spine of book, what does a page look like inside?

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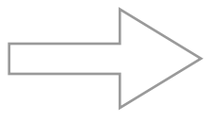

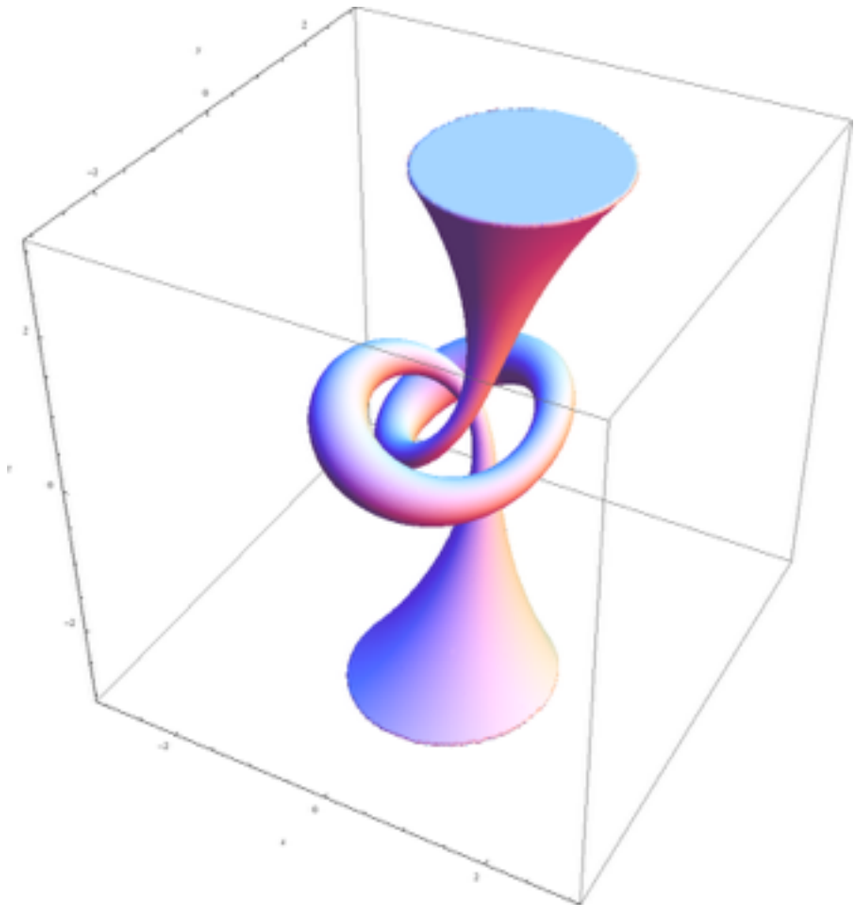

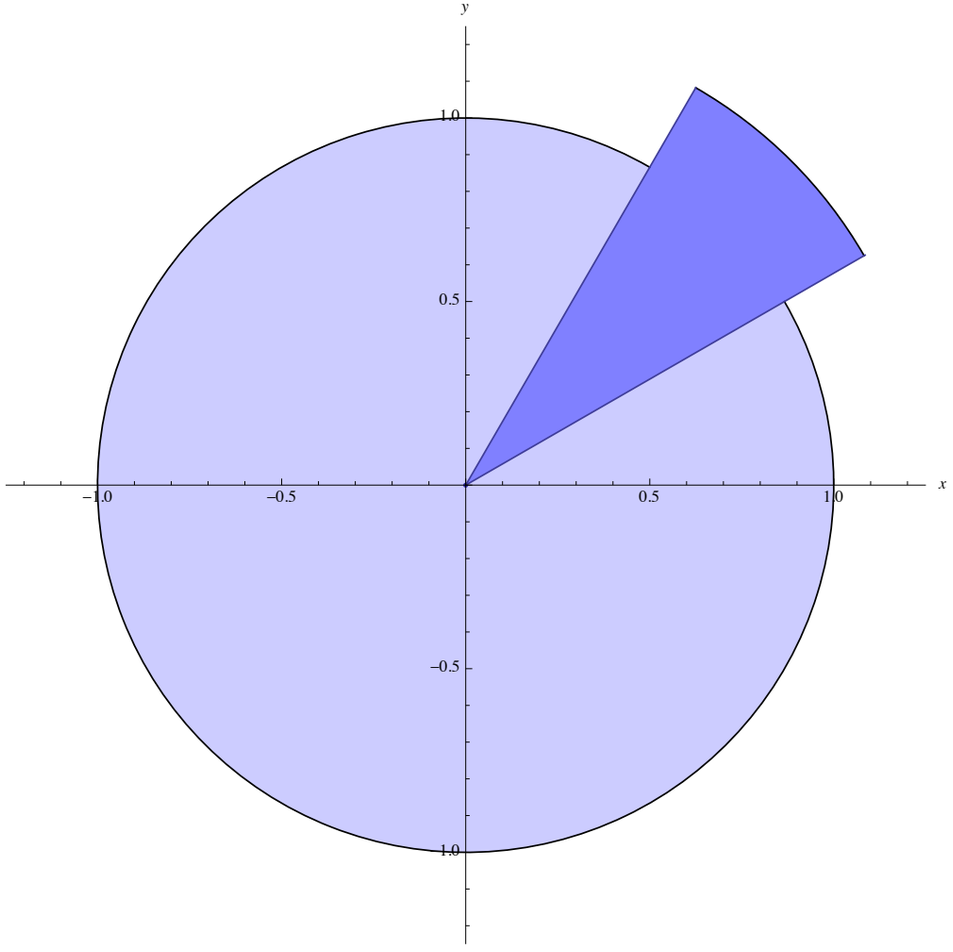
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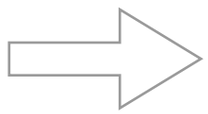

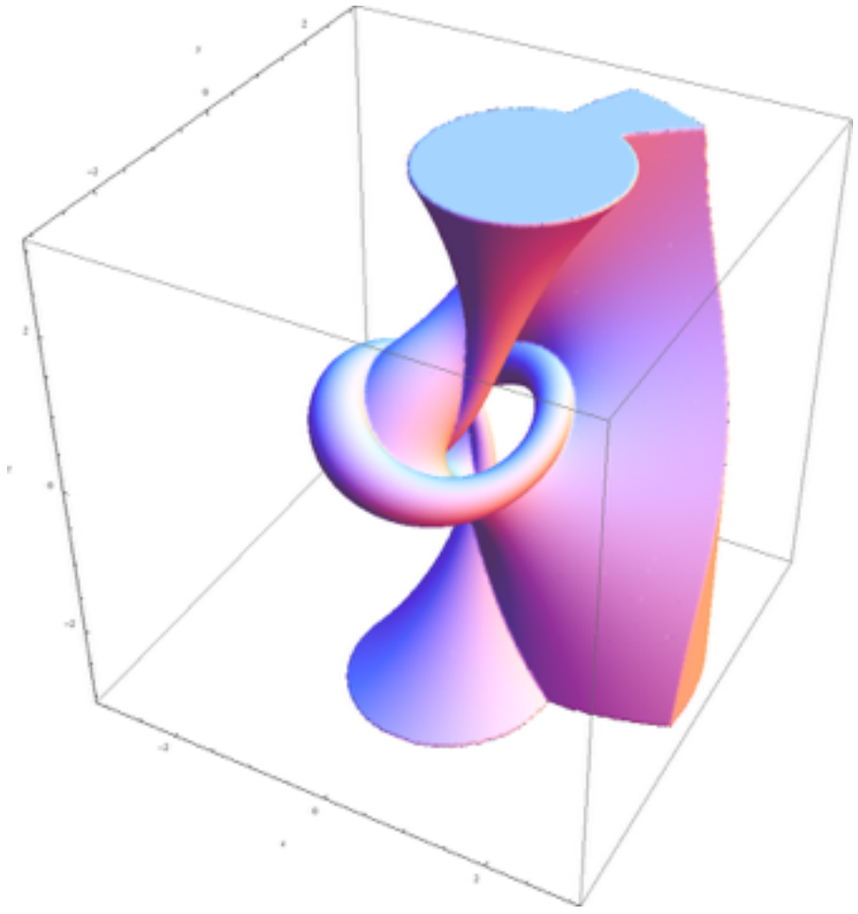

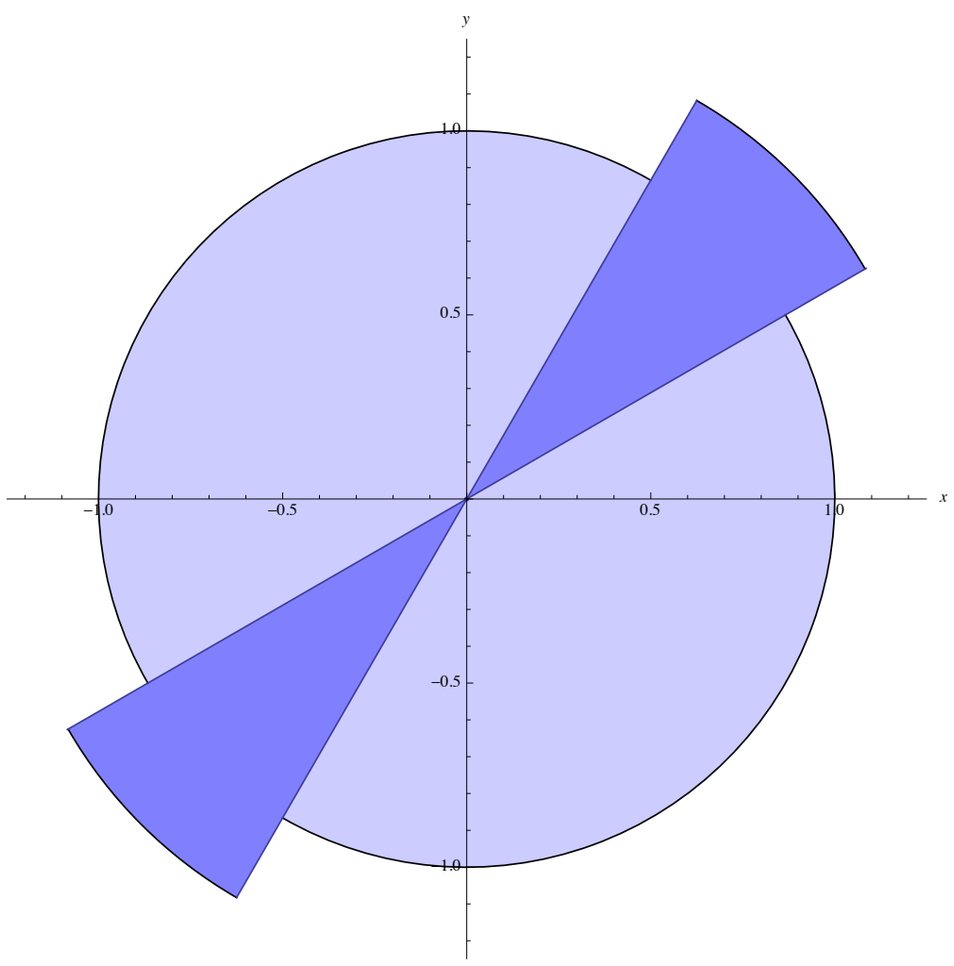
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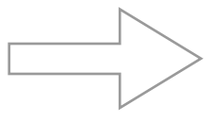

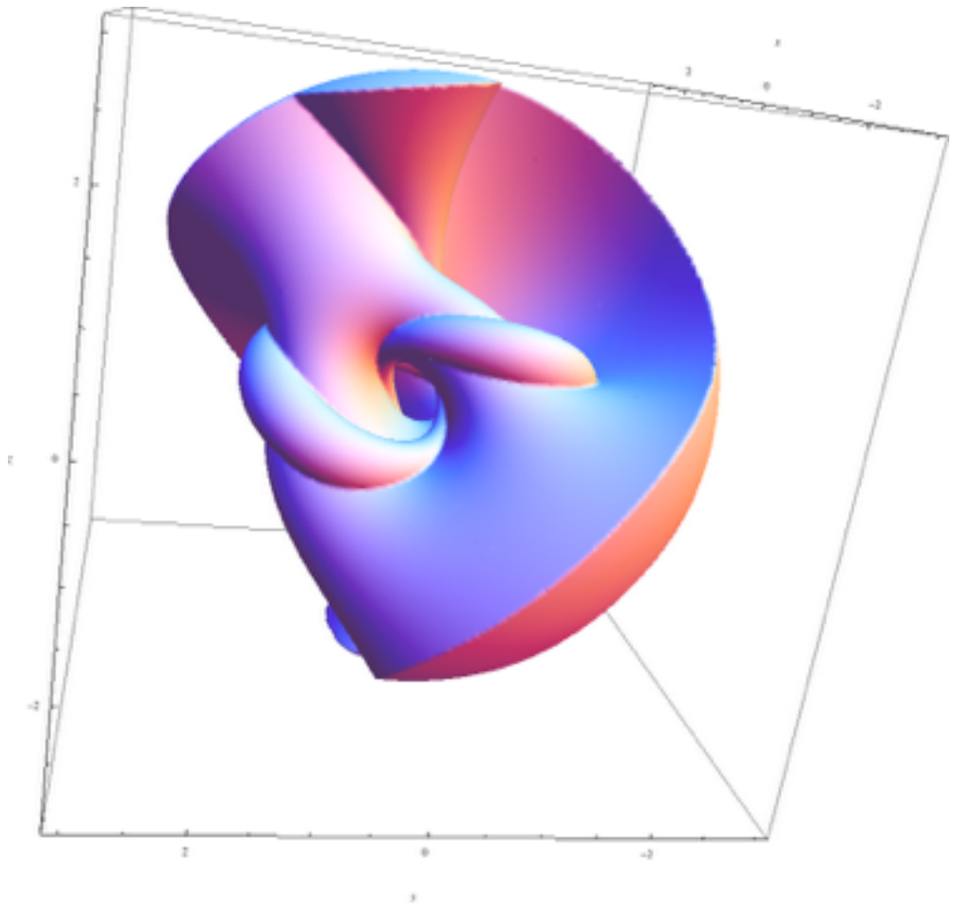

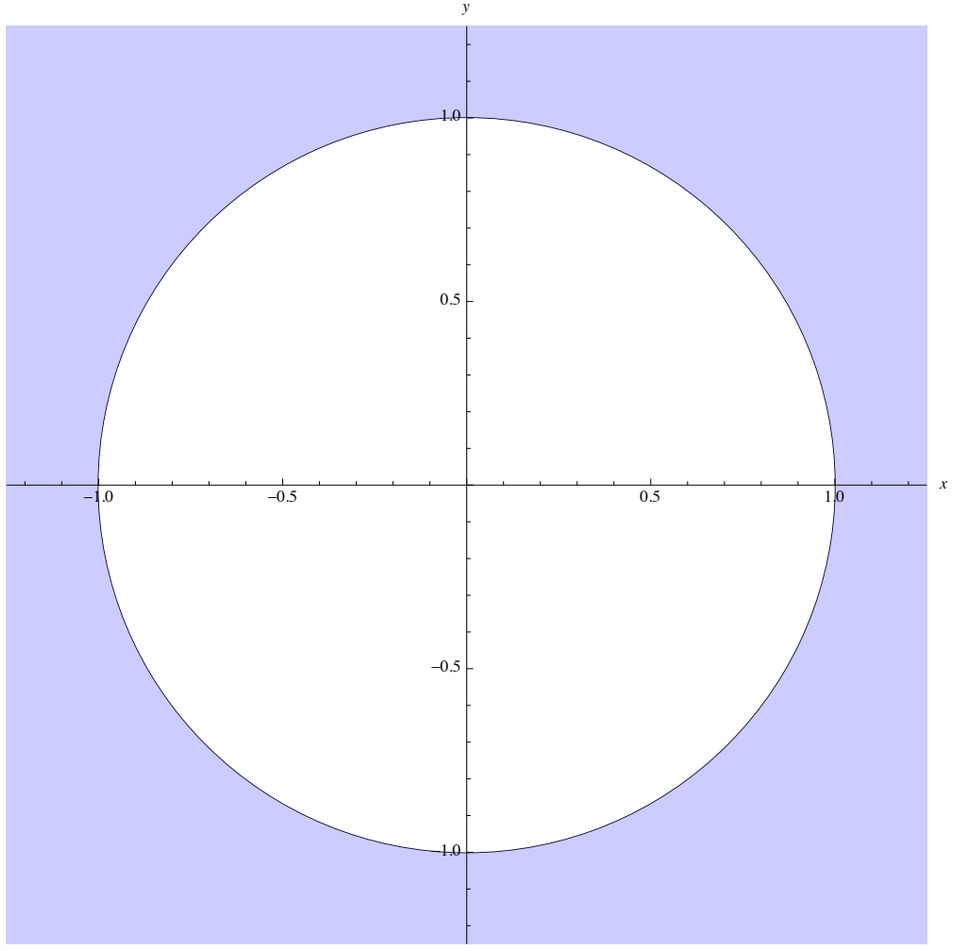
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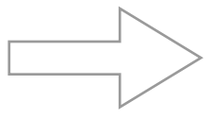
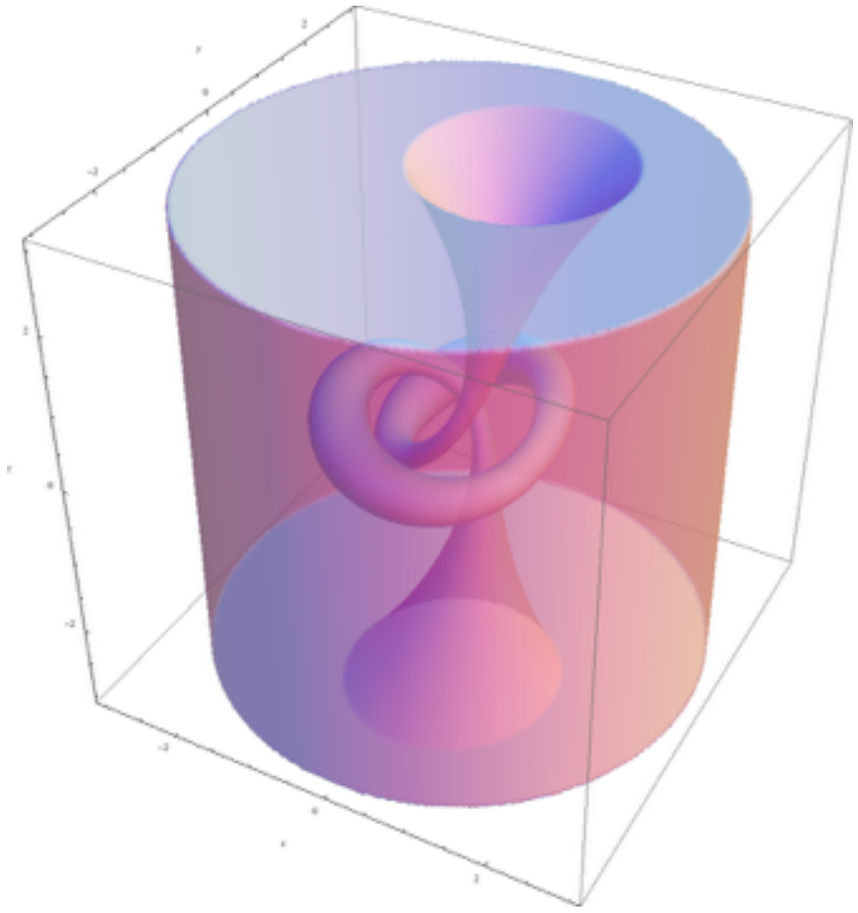
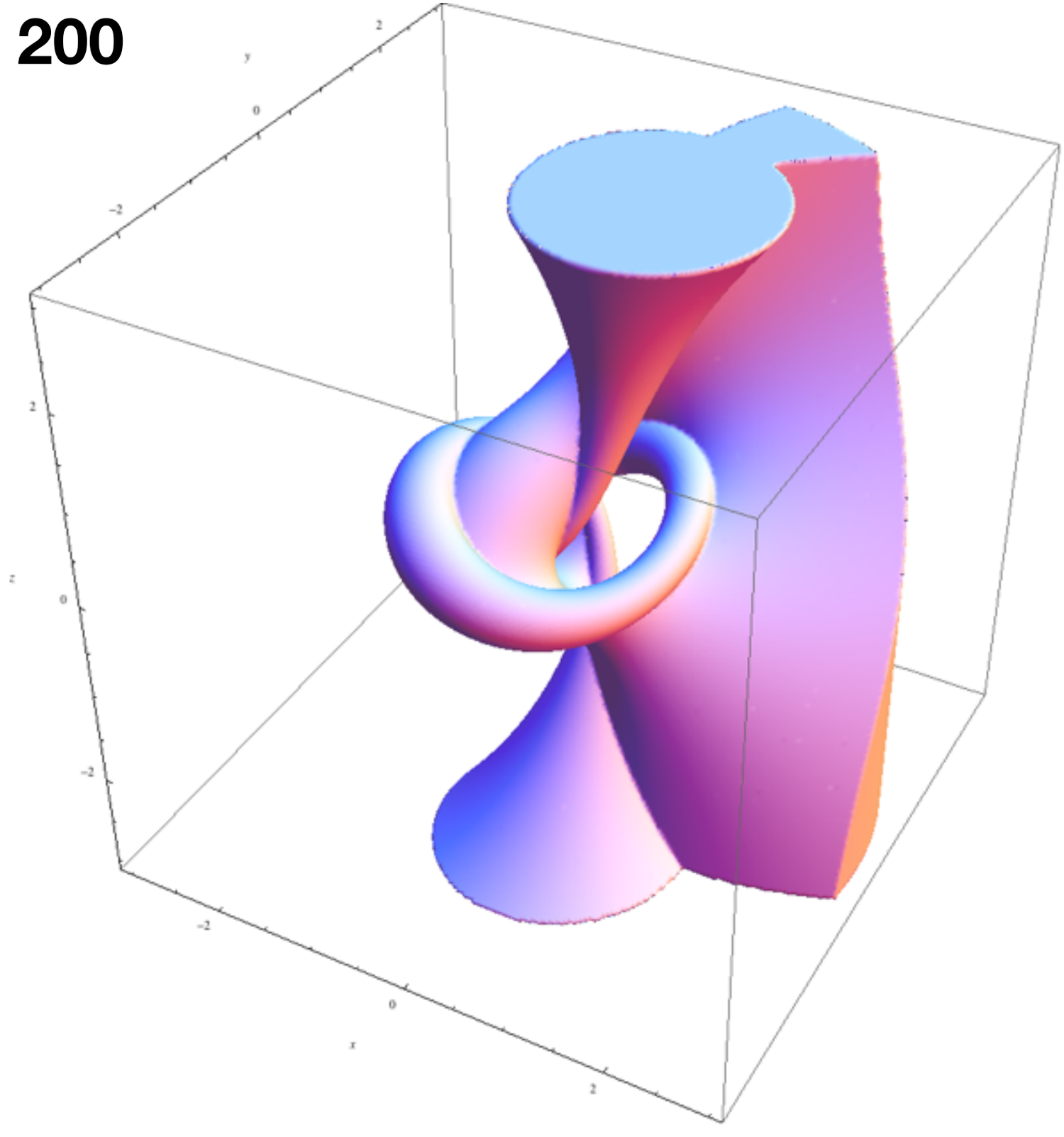




Generating 3D Models.

- **ST**ereo**L**ithography files (.stl) describe surface geometry of 3D objects
- Software: Mathematica, Blender, Meshlab, CAD, etc.
- **Resolution**: quality of model
  - more triangles + more vertices = higher resolution
  - Achieved in Mathematica via PlotPoints

200



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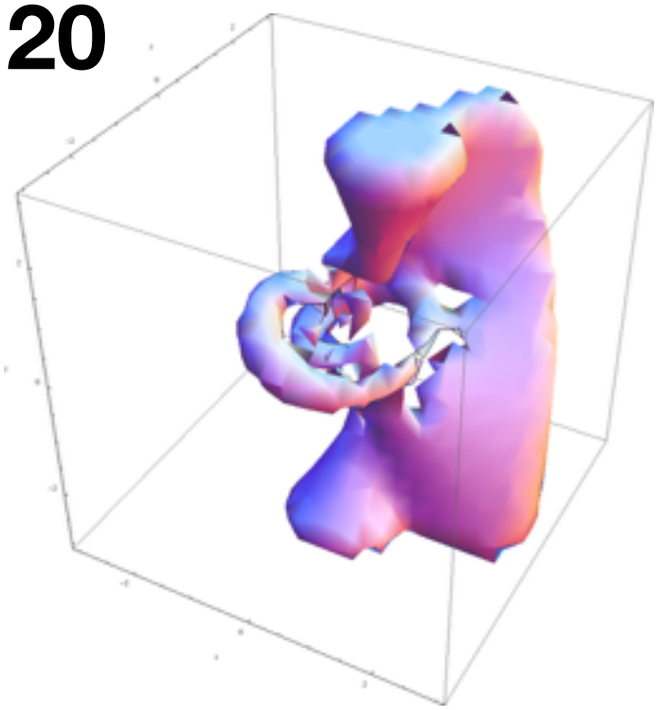

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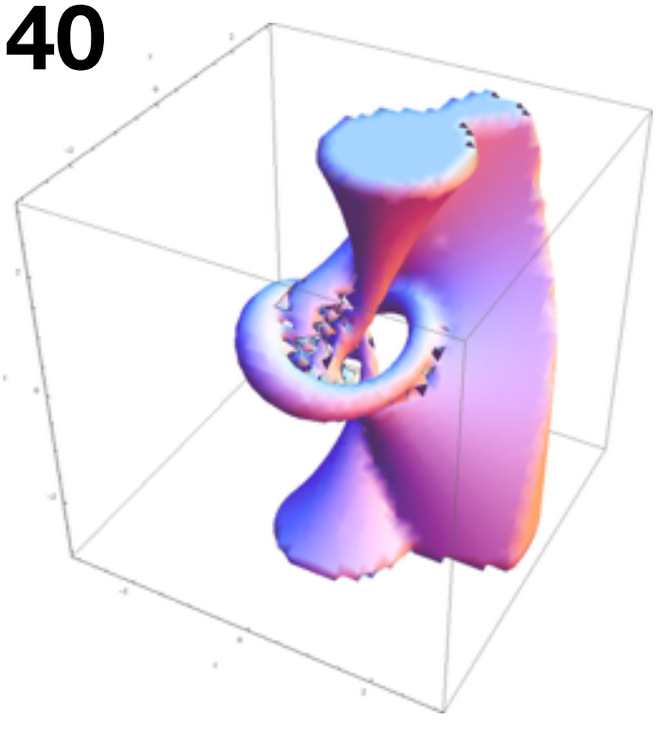


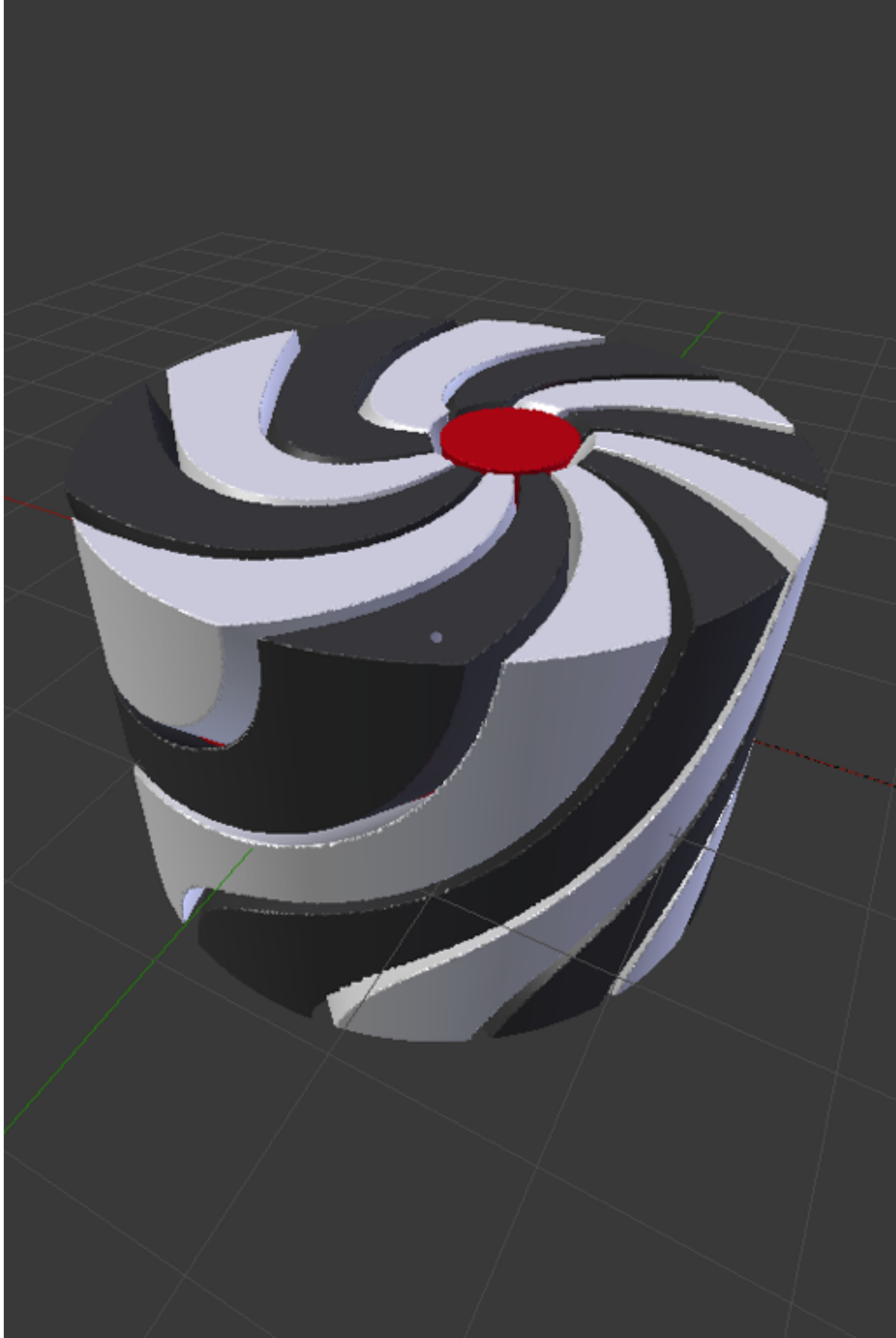
Generating 3D Models—Trefoil Knot + Page.
Created in Mathematica.

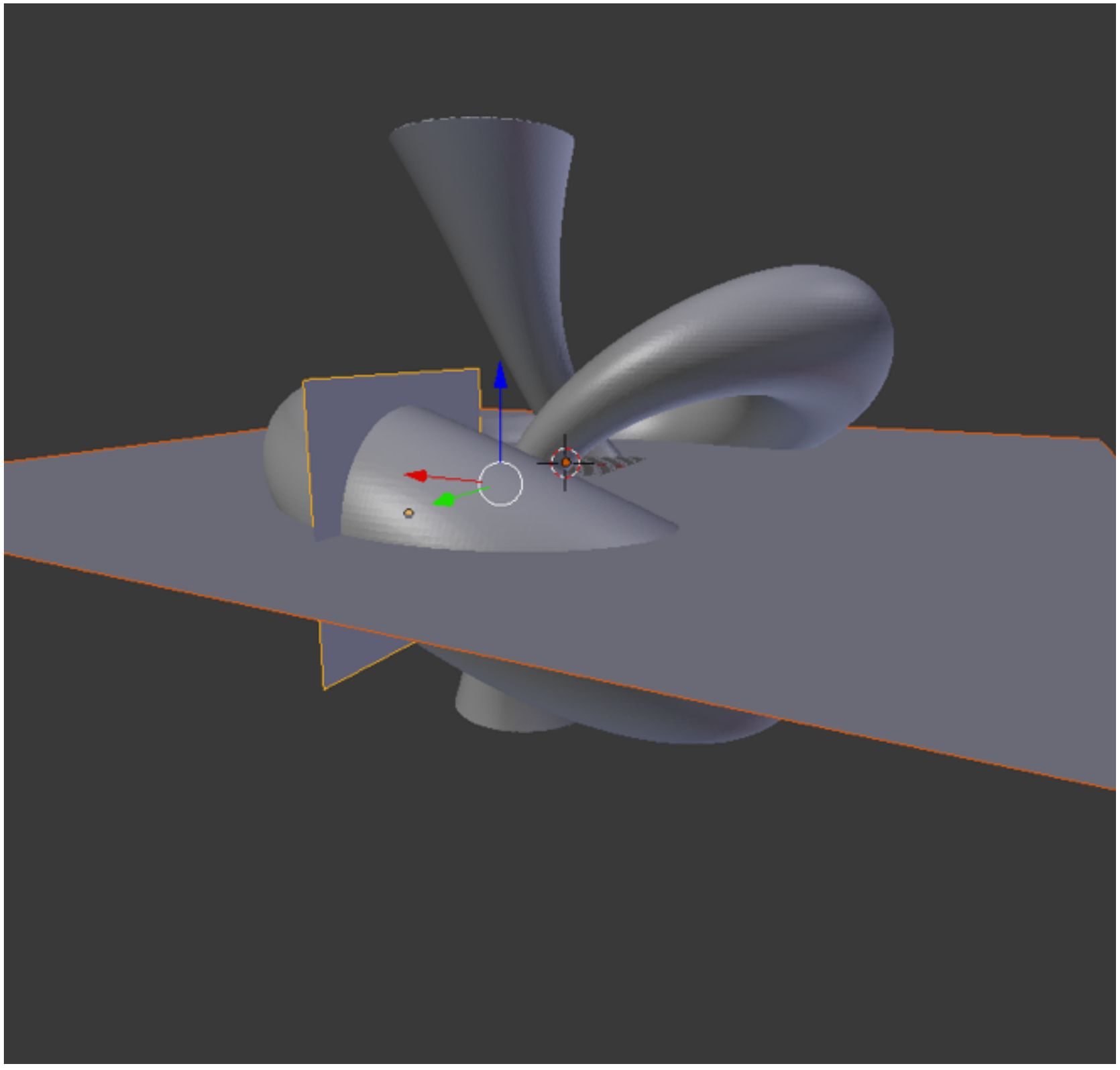
3D Mesh Manipulation.

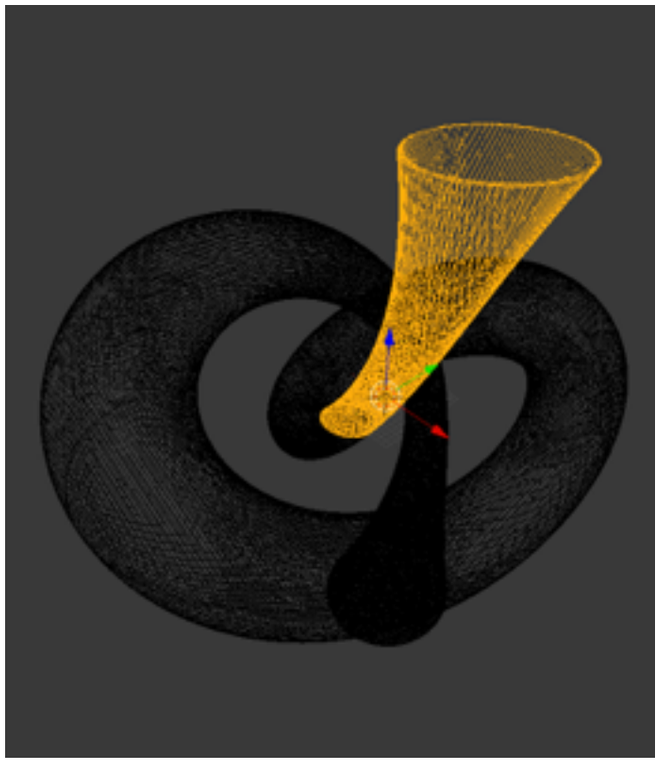
1. Cut in half
2. Account for connected pages 6 and 7
3. Cut trefoil knot to fit inside pages
4. Add numbers to each half page
5. Add holes to each piece

12+**12**+**2**+**2**+**6** = **34** pieces.

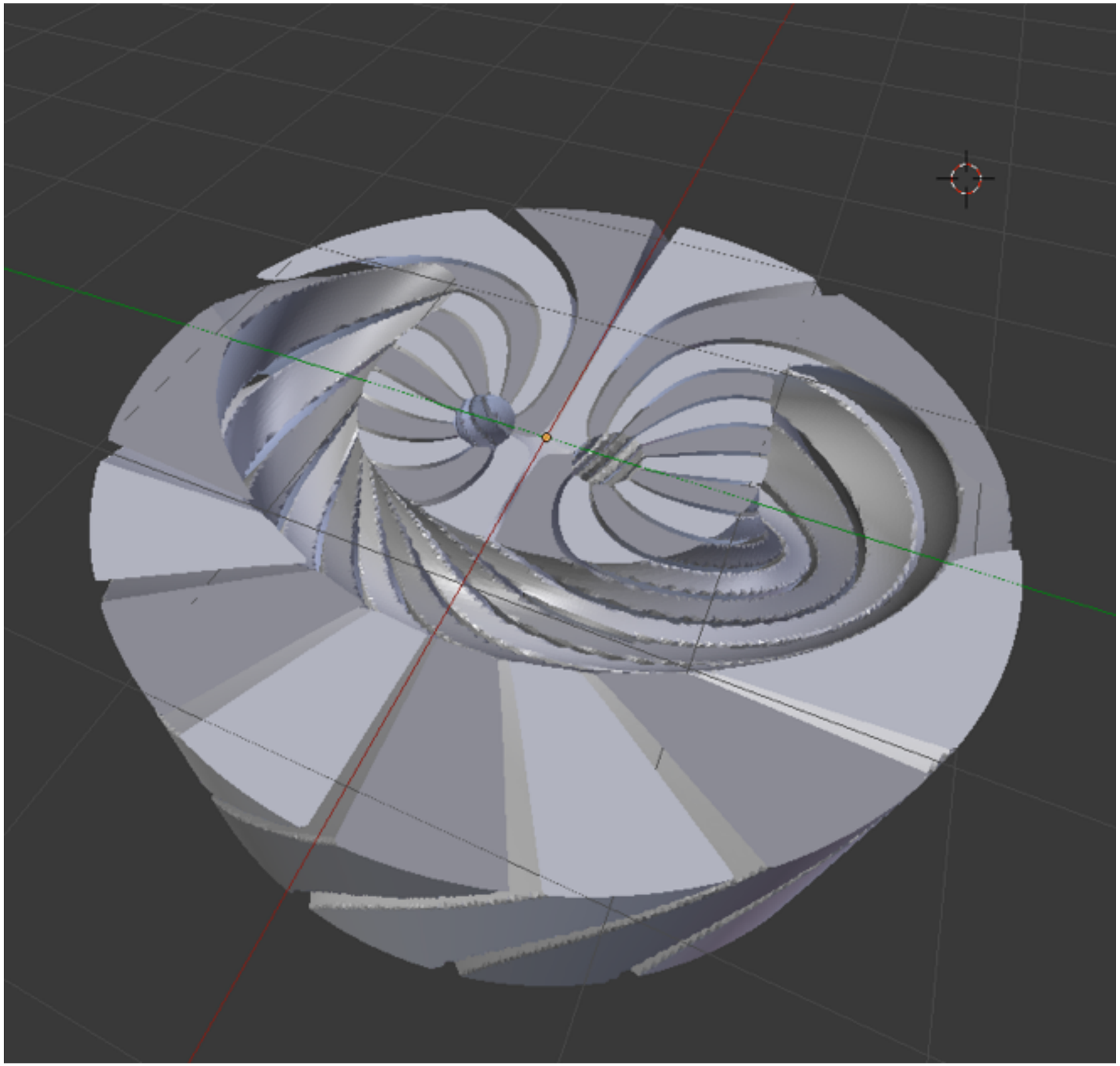
24+**24**+**6**+**6**+**10** = **70** holes = **70** magnets.

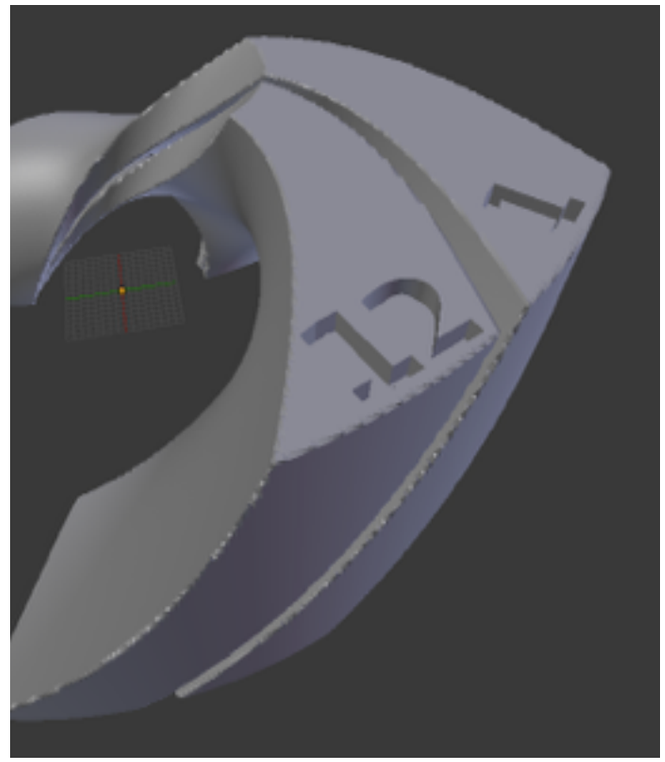


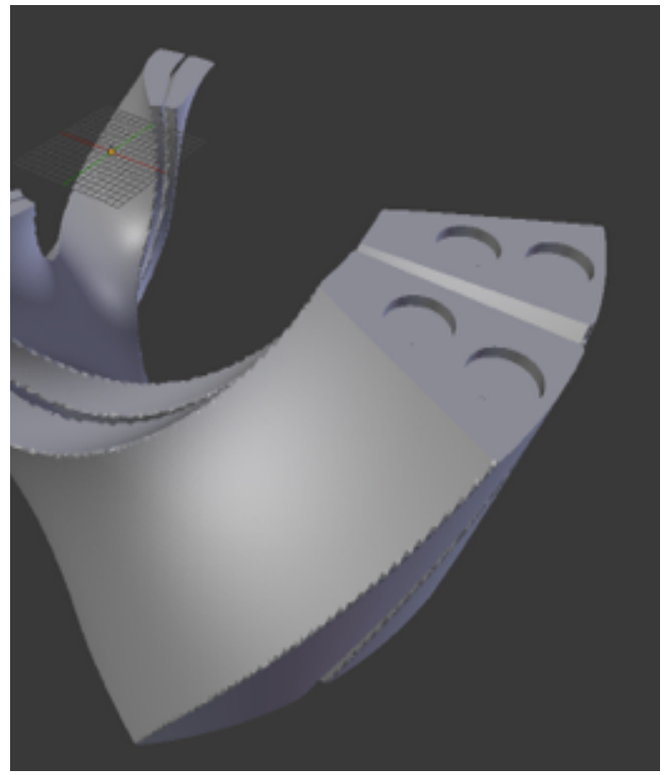




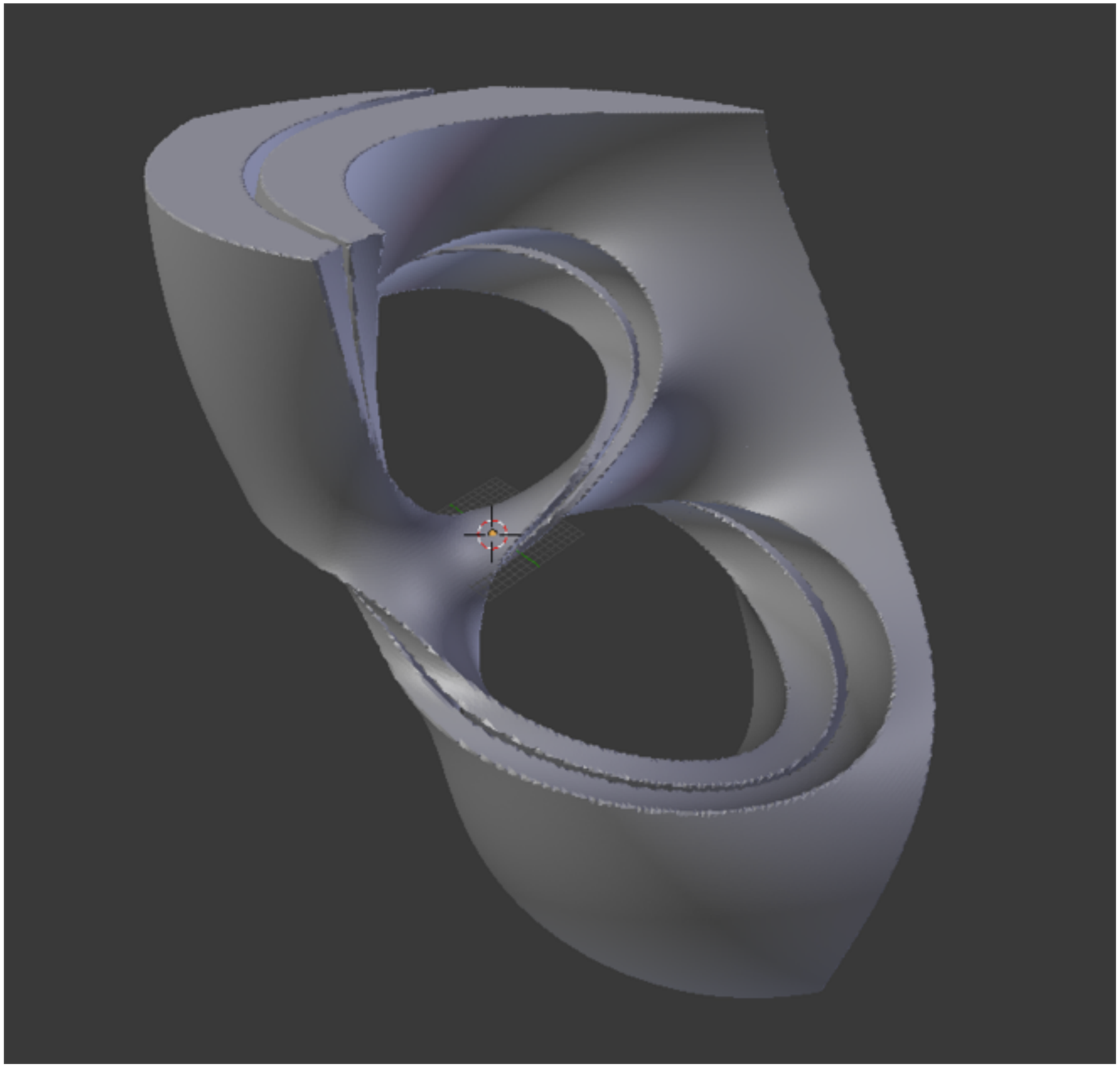
Blender Manipulations: Trefoil Knot.

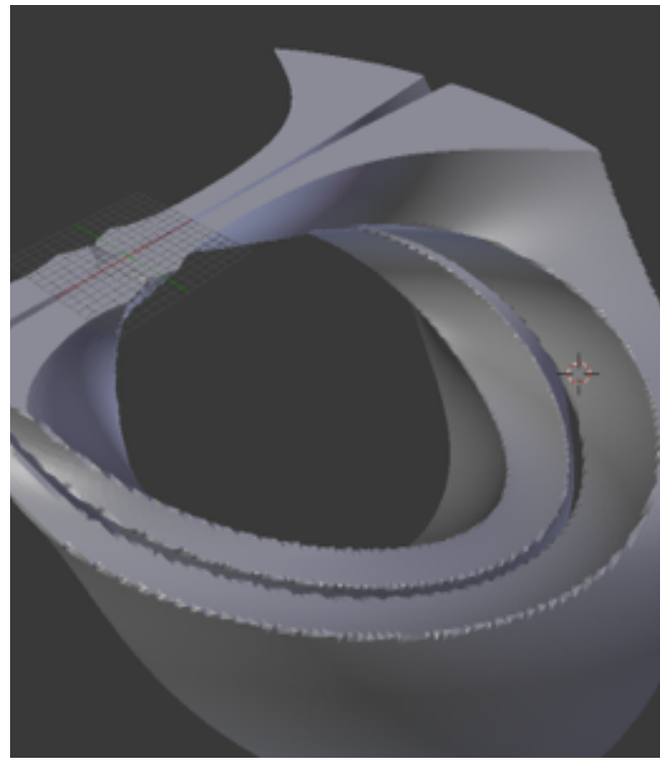


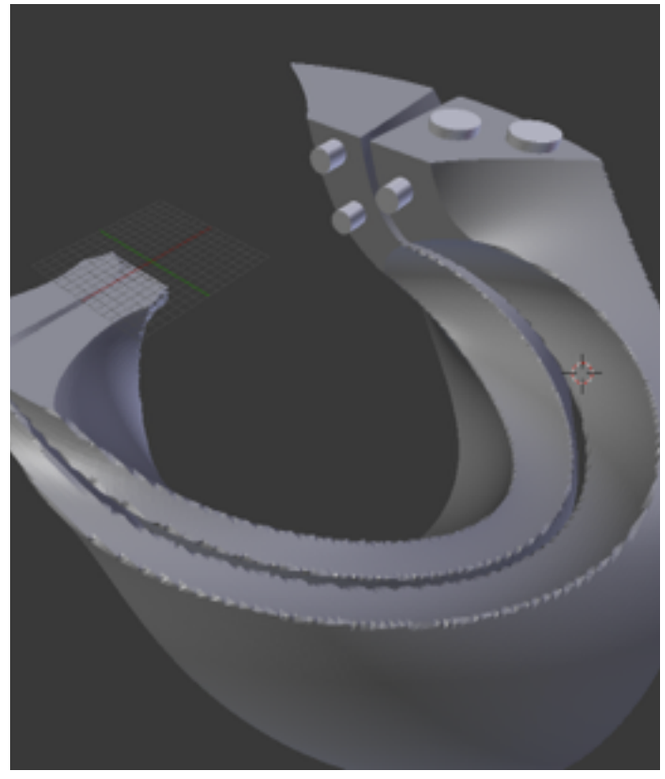




Blender Manipulations: 12 Pages.



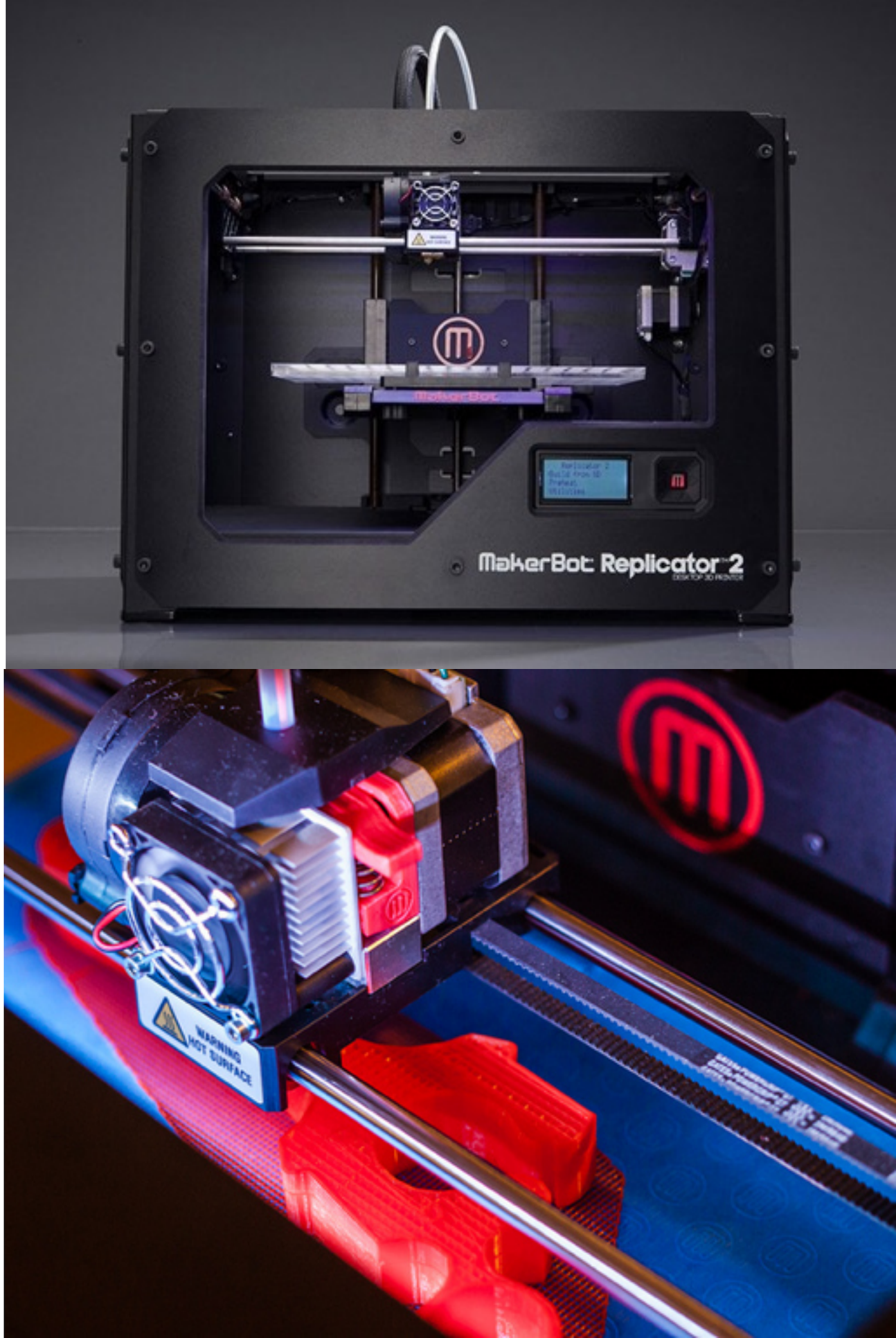




Blender Manipulations: Pages 6 and 7.

3D Printing.

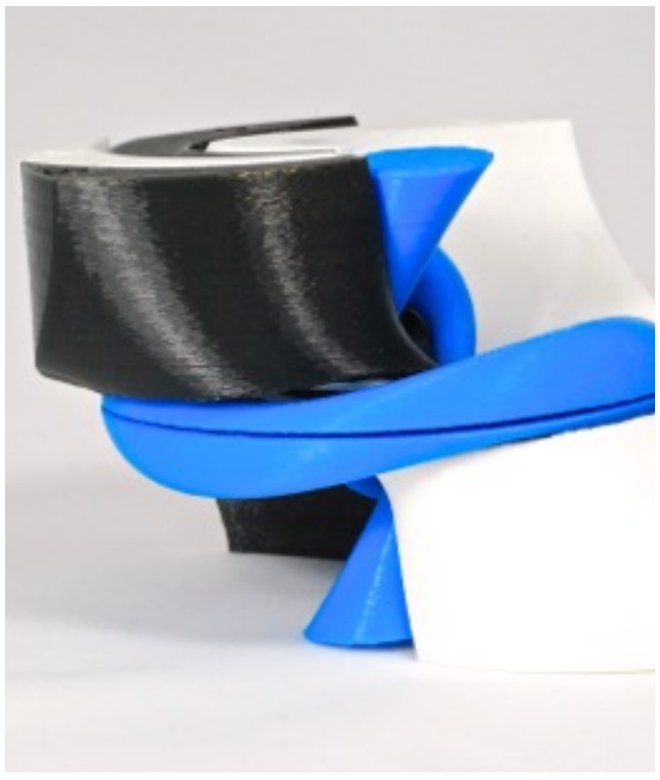
- MakerBot Replicator 2
- Controls:
  - **Raft** and **Supports**: on
  - Infill percentage (how solid/ hollow the object is): 15%
  - Layer Height: 0.2mm
  - Extruder temperature: 230° C, ~446° F
  - Number of shells (boundary layers): 3



Trefoil Knot
+ 12 Pages.

Trefoil Knot + 12 Pages.

Trefoil Knot + 4 Pages.



Trefoil Knot + 4 Pages.

Trefoil Knot + 6 Pages.

Trefoil Knot + 4 Pages.

Results.

- Handful of models posted on **Thingiverse**
  - MakerBot's website for digital design file sharing
  - Currently has 6500+ views and 1000+ downloads (Mar. 31, 2015)
- **Wolfram Community**
  - Over 5,500+ views
- Written about by **3dprint.com**

Final Remarks and Questions.

- **Tinkercad**: free 3D modeling program that runs in the browser (Chrome)—no downloading required!
  - Short tutorial (~10 minutes)
  - Easiest way to begin to model
- **Thingiverse**

Final Remarks and Questions.

Questions?

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References. (online)

- Mandalland Blogspot—Triangles
- Wikipedia—3D Printing
- Thingiverse—Stereographic Projection
- Makerbot
- Atomic Spin—Replicator 2 Issues

Full Photo Gallery available at
fredhohman.com/projects/trefoil-puzzle